Operations Research II
University of California, Berkeley
Spring 2006
Midterm 2

There are 2 questions for a total of 85 points. I have added some formulas on page 2 which you can use/quote in the exam at your own discretion.

1. **[10+10+10+10+10+10]** They ladies' room in Harry's bar has one toilet and, in accordance with local ordinances, also possesses a couch where **two** ladies may sit and wait their turn. Others who arrive when the couch and toilet are fully occupied must stand and wait their turn. Ladies spend an **average of 5 minutes** in the toilet, the actual time being exponentially distributed. On a given night ladies come to use the facility according to a Poisson process with a rate of **1 every 15 minutes**.

   (a) What are the states of this system and what is the steady state distribution?
   Hint: Be careful about the difference between the *mean* and the *rate*.

   (b) What proportion of time is the toilet busy?

   (c) What proportion of arriving customers will have to spend some time standing up? Would this conclusion change if the arrivals were no longer Poisson but still occurred at the same rate?

   (d) What is the expected time that any particular lady will have to spend standing up?

   The men's room at the bar contains one toilet and no couch. Men arrive at the men's room according to a Poisson process with average time of **10 minutes between** one arrival and the next. Each user of the toilet takes an exponential amount of time with **mean 3 minutes**. Arriving men who ~~find~~ find the toilet occupied wait; the queue spills out into the hall if necessary.

   (e) Harry will install a second toilet in the men's room is he is convinced that the expected time away from the bar is at least 5 minutes. How much time are men visiting the toilet currently spending away from the bar? What would the arrival rate to the men's room have to be to justify the extra toilet?

   (f) What is the probability that an arriving man will spend more than 10 minutes away from the bar?
   Note: If you are running short of time, you might want to leave part (f) until the end.

2. **[15 + 10]** Consider a single-server exponential system in which ordinary customers arrive at rate λ and have service rate μ. One of the customers in this system is Nicole Kidman who has a service rate μ_n. Whenever Nicole arrives, she goes directly into service (if anyone else is in service, then this person is bumped back into (the front of) the queue). When Nicole is not being serviced, she spends an exponential amount of time (with rate θ) out of the system.

   (a) Define an appropriate state space and set up the balance equations.

   (b) Find the probability than an ordinary customer is bumped at least n times.

Some formulas that may be helpful in the test:

For an M/M/1 queue:

$$P_0 = 1 - \frac{\lambda}{\mu}, \quad P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right), \; n \geq 1$$

$$L = \frac{\lambda}{\mu - \lambda}$$