IEOR 161 Operations Research II
University of California, Berkeley
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Midterm 1 Question 3

(a) (i) Let N(t) be the number of female customers when the first male customer arrives at time t.

$$E[N(t)] = \lambda t$$

$$E[\text{number of customers before first male customer}] = \int_{t=0}^{\infty} (\lambda t)\mu e^{-\lambda t} = \frac{\lambda}{\mu}$$

(ii) Let M be the Poisson process for men arrivals, and W be the Poisson process for women arrivals.

$$Pr\{M(1) = 0, W(1) = 0\} = Pr\{M(1) = 0\}Pr\{W(1) = 0\} = e^{-\lambda \cdot 1}e^{-\mu \cdot 1} = e^{-(\lambda+\mu)}$$

(iii) Let S_2 be the 2nd male customer arrival time.

$$E[S_2] = 1 + E[Y_1 + Y_2] = 1 + \frac{2}{\lambda}$$

(b) Let Y be the time it takes for the first man to arrive and X be that for the first woman to arrive. $Y \sim exp(\lambda)$, $X \sim exp(\mu)$

$$\begin{aligned}
Pr\{Y < t | Y < X\} &= \frac{Pr\{Y < t, Y < X\}}{Pr\{Y < X\}} \\
&= \frac{Pr\{Y < X < t\} + Pr\{Y < t, X > t\}}{Pr\{Y < X\}} \\
&= \frac{\int_{x=0}^{t} Pr\{Y < x\}\mu e^{-\mu x}dx + (1 - e^{-\lambda t})e^{-\mu t}}{\lambda/(\mu + \lambda)} \\
&= \frac{\int_{x=0}^{t} (1 - e^{-\lambda x})\mu e^{-\mu x}dx + (1 - e^{-\lambda t})e^{-\mu t}}{\lambda/(\mu + \lambda)}
\end{aligned}$$