

Operations Research II
University of California, Berkeley
Spring 2004
Midterm 1

3 questions (45 points) + 1 bonus question (10 points). Any score of 45 or above will be regarded as a perfect score.

1. **[10 points]** A doctor has scheduled 2 appointments, one at 1pm and the other at 1:30pm. The lengths of time that appointments last are independent exponential random variables with mean 30. Assuming that both patients are on time, find the expected amount of time the 1:30 appointment spends at the doctor's office.
2. **[10 points]**. Customers arrive at a two-server service station according to a Poisson process with rate λ . Whenever a new customer arrives, any customer that is in the system immediately departs. A new arrival enters service first with server 1 and then with server 2. If the service times at the servers are independent exponentials with respective rates μ_1 and μ_2 , what proportion of entering customers completes their service with server 2?
3. Men arrive at a store according to a Poisson process with rate λ (per hour), while women arrive according to a Poisson process with rate μ (per hour). Men and women arrive independently.
 - (a) **[5 + 5 + 5 points]**.
 - i. What is the expected number of customers you will see until you see the first male customer?
 - ii. What is the probability that the first customer (either male or female) arrives after $t = 1$ hour?
 - iii. What is the expected arrival time of the second male customer, given that the first male customer arrives after $t = 1$ hour?
 - (b) **[10 points]**. What is the probability that the first man arrives before time t , given that the first man arrived before the first woman?
4. **Bonus Question - [10 points]** A pipe-smoking man carries, at all times, 2 matchboxes, 1 in his left-hand pocket and 1 in his right-hand pocket. Each time he needs a match he is equally likely to take it from either pocket. Consider the moment when he first discovers that one of his matchboxes is empty. If it is assumed that both matchboxes initially contained N matches, what is the probability that there are exactly k matches in the other box, $k = 0, 1, \dots, N$?