IEOR160: Operations Research I

Midterm Exam
October 29, 2008

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Name: ______________________________ (please print)

SID: ______________________________

- Clearly state all the mathematical expressions that are needed to solve the problems. **No credit will be given to numerical answers without the proper setup.**
- Answer each of the following questions in the space provided. If you need more space to show major computations you performed to obtain your answer for a particular problem, use the back of the preceding page.
- You can quote and use any result stated in class or in the main body of the textbook as well as well-known general mathematical results but **no** references to other sources (including homework and textbook exercises) are allowed.
- Present your work in an organized and neat fashion.
- **Assume that all the functions are twice continuously differentiable.**
- Good luck!

Problem	1 (35)	2 (35)	3 (35)	Total (105)
Score				

A score of 100 would be considered as perfect.

Problem 1 (35 points)

A farmer wishes to fence in a rectangular pen for her animals. She only has 200 feet of fencing to use, but the back wall of the barn is 40 feet long and she can use it for part or all of one side of the pen.

a) Formulate a nonlinear programming problem so that the area of the pen will be as large as possible. Use only one variable in the formulation.
b) Write the KKT conditions for the problem.
c) Solve the system you got in (b).
d) Is the solution you got from (c) the optimal solution for the problem? **Explain.**

Problem 2 (35 points)

Consider the following problem:

$$\text{(P)} \quad z = \max \quad -x^2 - y^2$$
$$\text{s.t.} \quad (x-1)^2 - y^2 = b$$

For questions (a)-(c) assume that b=0.

a) Write the Lagrangian function for (P).
b) Determine **all** the points at which the gradient of the Lagrangian function is zero.
c) Given the fact that (P) has a global solution, find it.
d) Find $\partial z/\partial b$.

Problem 3 (35 points)

Read each of the following statements carefully to see whether it is *true* or *false*. **Justify your answers (no credit for answers without justification!)**

For the following question suppose **x** is an n-dimensional column vector and let ∇**f(x)** denote the gradient of a function **f** at **x** (presented as a row vector).

a) Suppose ∇**f(x*)d=0** for all n-dimensional column vectors d. Then **x*** is a local maximum point of **f**.

b) If ∇**f(x*)=0** and all leading principal minors of the Hessian of **f** are negative for all the points in $\mathbf{R}^n$, then **x*** is a global maximum point for **f**.
Note – there are no constraints in this question.

c) Consider:

(P1) **Max f(x)**
s.t. $g_i(x) \geq b_i$**, i=1,...m**

Assume **f** and g_i (i=1,…,m) are all concave functions. If **x*** satisfies the KKT conditions and the constraint qualifications, then **x*** is a global maximum point.

d) Consider:

(P2) **Max f(x)**
s.t. $ax \leq b$ (where **a** is an n-dimensional row vector)

Assume **f** is a concave function. If **x*** satisfies $ax^* < b$ and ∇**f(x*)=0**, then **x*** is a global maximum point.

e) Consider:

(P3) **Max f(x)**
s.t. $g_i(x) \leq b_i$**, i=1,...m**

Assume **f** is concave and g_i (i=1,…,m) are convex. Let **x*, y*** be two (global) optimal solutions for (P3). Then, **0.5x*+0.5y*** is also a (global) optimal solution for (P3).