Department of Industrial Engineering & Operations Research

IEOR160 Operations Research I

Exam 1 **10/17/2005**

Name: **Grade:**

1. **(15 points)** Determine whether the following statements are true or false and explain your answer.

   (a) T F A convex function has a single minimizer.

   (b) T F If f is a concave differentiable function, then the condition that the gradient of f is zero at $\bar{x}$ is a necessary and sufficient condition for $\bar{x}$ to be a maximizer of f.

   (c) T F If the objective function of an optimization problem is concave and its feasible region is defined by equality constraints, than it must be maximization problem.

   (d) T F If the gradient and the Hessian of f at $\bar{x}$ is zero, then $\bar{x}$ is a saddle point of f.

   (e) T F If $f(x) \leq f(y)$ for all y such that $||x - y|| \leq \epsilon$ for some $\epsilon > 0$, then f has a global minimum.

2. **(15 points)** Let $f : \mathbb{R}^n \mapsto \mathbb{R}$ be a concave function and $g : \mathbb{R} \mapsto \mathbb{R}$ be a nondecreasing concave function. Show that the composite function $g(f(x))$ is concave as well.

3. **(20 points)** Consider $f(x_1, x_2, x_3) = -4.5x_1^2 + cx_1x_2 - 2x_2^2 + cx_3 + d$ with $c \neq 0$.

(a) For which values of c and d is f a concave function?

(b) Determine all local and global maxima of f.

4. **(25 points)** Consider the following optimization problem:

$$
\text{(P)} \qquad \begin{aligned} z = \min\ & x^2 + y^2 \\ \text{s.t.}\ & (x-1)^2 - y^2 = 0 \\ & x, y \in \mathbb{R}. \end{aligned}
$$

(a) Write the Lagrangian function for (P).

(b) Determine all stationary points of the Lagrangian function.

(c) What is the optimal objective value z for (P)?

(d) Draw the feasible region and isocost (isoprofit) curves and illustrate the stationary points in part (b) using the gradients.

5. **(25 points)** JetGreen Airlines uses optimization methods to maximize its profitability. JetGreen wants to set fares for its business and economy classes for its daily flight between Oakland and New York. The demand as a function of price (p) is estimated to be $a_b - p$ for the business class and $a_e - p$ for the economy class. The cost of operating the flight has three components: The first component is a fixed dollar amount f, independent of the number of passengers (this is the cost of crew, insurance, regular maintenance, etc). The second component is a function cd^2 of the total number of passengers d on the flight (this is the fuel cost as a function of the estimated weight carried). The third component is the class-dependent cost of service, which is s_b dollars per business class passenger and s_e dollars per economy class passenger. If an airplane has k total seats, 15% of the total seats is allocated for the business class. If there is a need, the airline may upgrade an economy class passenger to the business class (at no cost to the passenger), but it cannot downgrade a business class passenger. A management policy states that at most 5% of the total passengers can be upgraded from the economy class to the business class.

   Develop a mathematical model to determine the fare for each passenger class so that the profit from the Oakland–New York flight is maximized. *Define your decision variables, state and explain the objective and constraints clearly.*