Department of Industrial Engineering & Operations Research

IEOR160 Operations Research I

Exam 1 **10/13/2004**

Name: **Grade:**

1. **(15 points)** Determine whether the following statements are true or false.

   T F A concave function cannot have a minimizer over an equality constrained feasible region.

   T F If the objective function of an optimization problem is convex and the feasible region is convex, then it is a minimization problem.

   T F If f is a continuously differentiable function, then all of its local maxima are among its stationary points.

   T F If x is a local maximum of a concave function, then there exists a direction vector d for which the directional derivative at x is negative.

   T F For a KKT point, if Lagrange multiplier of a constraint is zero, then the constraint is inactive at this point.

2. **(15 points)** Definition: A function $f : \mathbb{R}^n \mapsto \mathbb{R}$ is *strictly quasiconvex* on $S \subseteq \mathbb{R}^n$ if for each $x, y \in S$ the following inequality holds:

   $$f(\lambda x + (1 - \lambda)y) < \max\{f(x), f(x)\} \text{ for all } 0 < \lambda < 1$$

   Prove that if f is strictly quasiconvex on S, then a local minimum of f on S is also a global minimum of f on S.

3. **(20 points)** Consider $f(x_1, x_2, x_3) = -2x_1^2 + cx_1x_2 - 2x_2^2 - x_3^2$

(a) For which values of c is f a concave function?

(b) Determine all stationary points, as well as local and global maxima of f.

4. **(25 points)** Consider the following optimization problem.

$$
(P) \qquad \begin{aligned}
\max\ & 4x_1^2 + x_1x_2 + x_2^2 \\
s.t.\ & x_1^2 + x_2^2 \leq 20 \\
& x_1^2 - x_2 \geq 0 \\
& x_2 \geq 0
\end{aligned}
$$

(a) Write the KKT conditions for (P).

(b) Determine whether $(x_1, x_2) = (2, 4)$ and $(x_1, x_2) = (-2, 4)$ are KKT points for (P).

(c) Check whether dual feasibility conditions hold at points $(2, 4)$ and $(-2, 4)$ graphically (Draw the feasible region and illustrate the gradients).

5. **(25 points)** A manufacturing firm produces 4 different products ($i = 1, 2, 3, 4$). The raw material that is used in all of these products is in short supply and at most r units can be purchased at \$$p$ per unit. The selling price of product i is \$$s_i$ per unit. Furthermore each unit of product i uses a_i units of of the raw material. The variable cost, excluding the raw material cost, of producing x_i units of product i is $k_i x_i^2$, where $k_i > 0$ is known. Develop a mathematical model to determine the production quantity of each product so that the total profit from this operation is maximized. *Define your decisions variables, state and explain the objective and constraints clearly.*