Solution to Midterm 1

1. K = 250; λ = 2000; i = 0.40; c = 2.00

(a)

$$Q = \sqrt{\frac{2(250)(2000)}{0.40(2)}} = 1118$$

but maximum shipment is 1000, so Q = 1000.

(Q > truck capacity implies that inventory is cheap, so we want to fill the truck. But it's not sensible to send more than 1 truck at a time. To convince yourself of this, draw the inventory curves for both scenarios and compute their respective annual costs.)

This quantity will satisfy demand for 6 months, so it may need to be reduced if the pepperoni has a limited shelf - life.

(b) The store needs to order when she has a 1-week supply remaining, or when the inventory falls to 2000/52 = 38.5 units (assuming demand is perfectly constant and delivery is perfectly reliable).

(c) Annual revenue = 3(2000) = 6000

Annual costs (obtainable using the annual cost formula for the EOQ model):

Purchasing costs = 2(2000) =	4000
Setup costs = 250(2000/1000) =	500
Inventory costs = 0.40 (2) (1000/2) =	400
Net profit =	1100

2.

(a)

P_1 = 200 * 8 * 300 = 480,000 / year

P_2 = 480,000 / year

$$\sum_{i=1}^{2} \frac{\lambda_i}{P_i} = \frac{200{,}000}{480{,}000} + \frac{100{,}000}{480{,}000} = 0.625 \le 1 \Rightarrow \text{ok}$$

$$h_i^{'} = cI(1 - \frac{\lambda_i}{P_i}) \Rightarrow$$

$$h_1^{'} = 1.5 \times 0.2 \times (1 - \frac{200{,}000}{480{,}000}) = 0.175, \; h_2^{'} = 1.6 \times 0.2 \times (1 - \frac{100{,}000}{480{,}000}) = 0.253$$

$$T^* = \sqrt{\frac{2(50+40)}{0.175(200{,}000) + 0.253(100{,}000)}} = 0.546 \text{ year}$$

$$T_{\min} = \frac{0.0021 + 0.0017}{1 - (\frac{200{,}000}{480{,}000} + \frac{100{,}000}{480{,}000})} = 0.01 \le T^* \Rightarrow 0.546 \text{ is the optimal solution}$$

(b) $Q_{origin} = T^* \times \lambda_1 = 0.0546(200{,}000) = 10{,}924$ units,
$Q_{cheese} = T^* \times \lambda_2 = 0.0546(100{,}000) = 5{,}462$ units.

3.
Parameters:

$$\lambda = 6000$$
$$K = \$25$$
$$I = 0.5$$

All units discount:

$$Q_3 = \sqrt{\frac{2(25)(6{,}000)}{0.5(4.5)}} = 361.15 < 6000 \Rightarrow \text{not feasible}$$
$$Q_2 = \sqrt{\frac{2(25)(6{,}000)}{0.5(4.75)}} = 355.4 < 3000 \Rightarrow \text{not feasible}$$
$$Q_1 = \sqrt{\frac{2(25)(6{,}000)}{0.5(5)}} = 346.4 \Rightarrow \text{ok}$$

Total costs: at solution and break-points:

$$G(346) = 10 \times \frac{6{,}000}{346} + \frac{0.5(5)(346)}{2} + 6000 \times 5 = 30{,}606$$
$$G(3{,}000) = 10 \times \frac{6{,}000}{3{,}000} + \frac{0.5(4.75)(3000)}{2} + 6000 \times 4.75 = 32{,}083$$
$$G(6{,}000) = 10 \times \frac{6{,}000}{6{,}000} + \frac{0.5(4.5)(346)}{2} + 6000 \times 4.5 = 33{,}760$$

So 346 is the optimal solution.

4. $\lambda = 250{,}000$
$\mu = 1000$ (for demand during lead time)
$\sigma = 200$ (for demand during lead time)
K=200

(a) Type 1 service, $\alpha = 0.99$
Q=EOQ=$\sqrt{\frac{2K\lambda}{Ic}} = 14907$
z=2.33 (from Table 4)
$R = \sigma z + \mu = 200(2.33) + 2000 = 2466$
Hence (Q,R)=(14907,2466)

(b) Type 2 service, $\beta = 0.98$
Q=EOQ=14907

$$\frac{n(R)}{Q} = \frac{\sigma L(z)}{Q} = 1 - .98 = .02$$

L(z)=1.49

z=-1.46

Note: I gave full credit as far as you are using the right model and L(z) is correct.

$$R = \sigma z + \mu = 200(-1.46) + 2000 = 1708$$

(c) If using the newsvendor model:

$$c_0 = IcQ / \lambda = 0.3 * 2 * 14907 / 250000 = 0.05376$$

$$c_u = 100 * 0.03 = 3$$

$$F(R) = \frac{c_u}{c_u + c_0} = 3 / 3.0596 = 0.9805 \text{ (type 1 service level)}$$

z=2.06

$$R = \sigma z + \mu = 200(2.06) + 2000 = 2412$$

May also use the formula on page 269 of the textbook (equation (1) and (2)) and find the solution iteratively as in problem 3 of homework 2.