Solutions for Midterm 1

1. Basic info:
λ=1,000,000
I = 0.50

Supplier A: K = 200
c_1 = 2.10 for Q up to 10,000
c_2 = 2.00 for Q ≥ 10,000

Supplier B: K = 50
Shipping costs create an incremental units discounted price structure
c_1 = 2.10 (including shipping) for Q up to 2000
c_2 = 2.00 (also including shipping) for more than 2000
Notice that by paying \$0.10 per unit on the first 2000 units, we have covered the \$200 shipping cost per truckload

For supplier A:

$$Q_1 = \sqrt{\frac{2(200)(1{,}000{,}000)}{0.5(2.00)}} = 20{,}000 \text{ feasible so stop}$$

For supplier B:

$$Q_1 = \sqrt{\frac{2(50)(1{,}000{,}000)}{0.5(2.10)}} = 9759 \text{ not feasible}$$

$$Q_2 = \sqrt{\frac{2(1{,}000{,}000)[50 + 0.10(2000)]}{0.5(2.00)}} = 22{,}361$$

This is feasible with respect to the discount structure
but this won't fit into the truck so use $Q = 20{,}000$

Notice now that we can get the same unit cost from both suppliers (ignoring the transportation costs) and the order quantities are identical. Supplier A has a total setup cost of \$200 per shipment whereas it is \$250 (including transportation) for Supplier B, so Supplier A is cheaper. Order 20,000 at a time from Supplier A. (Using this logic, there was no need to compute total annual costs.)

2 (a) This is a newsvendor model. The decision concerns tokens for the current month, BEFORE the fare increase takes place. The fact that the fare is rising to \$2.50 is irrelevant in the calculations because the old tokens cannot be used after the end of the current month.

Demand is Normal with mean = 80, std. dev. = 20

$c_u = 2.00 - 1.80 = 0.2$ (if we run out of discounted tokens before the end of the month, we have to pay \$2.00 instead of \$1.80)

$c_o = 1.80$ (if we have leftover discounted tokens at the end of the month, they are essentially worthless)

$$F(S) = \frac{0.2}{0.2 + 1.8} = 0.1$$

$$z = -1.28$$

$$S^* = 80 - 1.28(20) = 54.4$$

purchase 55 - 10 = 45

(b) This is an infinite horizon model without setup costs (under the assumption that it is easy for the student to go to the subway station at the beginning of each month).

Demand is Normal with mean = 20, std. dev. = 5

shortage cost $\pi = 2.00 - 1.75$

holding cost for one month $h = \frac{0.24}{12}(1.75) = 0.035$

$$F(S) = \frac{0.25}{0.25 + 0.035} = 0.8772$$

$$z = 1.17$$

$$S^* = 20 + 1.17(5) = 25.85$$

Because unused rides carry over from one month to the next, the student should purchase (order) UP TO 26. This presents a slight dilemma if she has more than 6 on hand at the end of the month. But the opportunity cost of purchasing a discount rideearly is only \$0.035 per month, whereas she can get a discount of \$0.25, so she can buy a token about 7 months early and still be ahead. So if she has more than 6 on hand at the end of the month, she should buy 20.

3. The most straightforward analogy arises when there is only one type of cookie. In this case, the EPQ model is appropriate. The setup cost is the cost of the energy to pre-heat the oven. The holding cost relates to cookies getting cold and stale. (Of course, the customers "pay" this part of the costs, but the cookie store should be concerned about it.) Finally, there is a finite production rate, and demand is fairly constant because "customers arrive steadily through the store's business hours."

If there are several varieties and you can bake them together in the oven, then the EPQ continues to be applicable. If, however, it's necessary to bake a batch of one variety before switching to another, the situation becomes more like the product cycling problem…..BUT it's not always necessary to perform a setup (i.e., turn the oven off and pre-heat it again) before you change to another cookie variety, which differs from the assumptions of the product cycling problem.

Some of you suggested other methods that would help to synchronize the production of cookies (to save energy), such as the (S,T) model, and credit was also given for such answers according to how well you explained and justified the approach.

Many points were taken off if you only described a model but did not explain the connection between the cookie scenario and your selected model.

4. Basic data:

λ=2000
K = 10
c=20
I = 0.3
$\mu_{\text{daily}} = 8$
$\sigma_{\text{daily}} = 3$

(a) The easiest way to solve this problem is to take Q as the standard delivery quantity of 20 boxes. Under this assumption, we have:
$p = 5(10) - 20 = 30$ (shortage cost is $30 for the equivalent of one box)

$\mu_{\text{LT}} = 2(8) = 16$

$\sigma_{\text{LT}} = \sqrt{2}(3) = 4.24$

$$F(R) = 1 - \frac{0.3(20)(20)}{30(2000)} = 0.9980$$

$z = 2.88$

$R = 16 + 2.88(4.24) = 28.21$

If you chose to find the EOQ and iterate, then you would find Q = 83 (approx), z = 2.40 (approx) and R = 27 (approx).

If you decided to use (S,T), you would have found T = 10 days (approx), z = 2.41 (approx) and $S = 8(10+2) + 2.41\sqrt{12}(3) = 121$ (approx).

(b) The fill-rate model.is appropriate here. If the paper is in stock 99.9% of the time, then 99.9% of arriving customer will have paper available. We have:

$\beta = 0.999$

If $Q = 20$, we want $n(R) = \sigma_{LT} L(z) = 20(1 - 0.999) = 0.2$

$$L(z) = \frac{0.2}{4.24} = 0.0047$$

$z = 2.23$

$R = 16 + 2.23(4.24) = 25.46$

Of course, if you used a different value of Q, your solution would differ.

If you iterated, you should have arrived at Q = 83 (approx) and R = 23 (approx).

(c) This scenario calls for Type 1 service. We want to have at most one stockout occasion per year. The value of α depends upon the value of Q that you assumed, because the choice of Q affects the number of order cycles per year. For Q = 20, we would have 2000/20 = 100 orders per year, and we can allow a shortage in 1 order cycle, so we want $\alpha = 0.99$. If you used Q = 80, then we would have 2000/80 = 25 orders per year, and we would want $\alpha = 0.96$.

For $\alpha = 0.99$, we get z = 2.33 and R = 16 + 2.33(4.24) = 25.88.

(d) In this scenario, we have periodic review and a positive lead time, so we need a model that will accommodate these factors. This makes the (S,T) model most suitable. (The model for (s,S) that we discussed in class assumes the lead time is zero.)

The solution depends upon whether you want to account for the setup cost ($10), or whether you think that reviewing the inventory daily (and possibly ordering) is acceptable. If you consider the setup cost, then T = 10 days (approx) as shown in part (a). If you want to allow daily review, then T = 1. Calculations are shown below for T = 1 day and τ = 1 day.

$$F(S) = 1 - \frac{0.3(20)(1/250)}{30} = 0.9992$$

$z > 3$; but the table in the book doesn't go beyond $z = 3$, so using $z = 3$, we get:

$S = (1+1)(8) + 3(4.24) = 28.72$

5. Many different answers were acceptable for this problem, but points were taken off if you considered only profit margin (or revenue) per unit or demand alone without explaining why you chose that metric. Typically, it is better to consider either profit or revenue per unit multiplied by demand because the product of these two factors provides a better indication of the importance of the product. In addition, more points were given for clearer and more complete explanations and justifications. (Some answers were quite brief and difficult to interpret.)