Midterm - Solutions

1. The effective monthly rate is 1%. EAR $= (1.01)^{12} - 1 = 0.126825$. Since this rate is greater than the YTM, you will prefer to sell the bond.[1] The bond is at par which implies that the price of the bond is \$10,000. Thus, you will be receiving $\$10,000$ at the end of each year. The total number of bonds is $2038 - 2009 + 1 = 30$. The present value at the beginning of 2009 is:

$$PV_{bonds} = 10,000\left(\frac{1 - 1.126825^{-30}}{0.126825}\right) = \$76655.48.$$

The PV of the first year's salaries is:

$$PV_1 = 8000\left(\frac{1 - 1.01^{-12}}{0.01}\right)$$

Similarly, the PV of the second year's salaries at the beginning of the second year is:

$$PV_2 = 8000 \cdot 1.06\left(\frac{1 - 1.01^{-12}}{0.01}\right)$$

Similarly,

$$PV_3 = 8000 \cdot 1.06^2\left(\frac{1 - 1.01^{-12}}{0.01}\right)$$

$$PV_4 = 8000 \cdot 1.06^3\left(\frac{1 - 1.01^{-12}}{0.01}\right)$$

and so on..

Therefore, these PV's form a growing annuity (due) where the growth rate is 0.06 and $C = 8000\left(\frac{1 - 1.01^{-12}}{0.01}\right)$:

$$\text{PV}_{salary} = \frac{\left[8000\left(\frac{1 - 1.01^{-12}}{0.01}\right)\right]}{0.126825 - 0.06}\left(1 - \left(\frac{1.06}{1.126825}\right)^{30}\right) \cdot 1.126825 = 1,275,723.67$$

Total PV $= \text{PV}_{bonds} + \text{PV}_{salary} =$ \$1,352,379.15.

2. Let's first find the required rate of return.

$$11 = \frac{D_1}{r - g} = \frac{1.10}{r - 0.10} \Rightarrow r = 0.20.$$

Then, the NPV of the first project is:

$$NPV_1 = -1000 + 500\left(\frac{1 - 1.20^{-10}}{0.20}\right) = 1096.24$$

[1] If you sell the bond, you will receive the price of the bond. The price is the PV of the future payments discounted at the YTM. If you hold the bond, you will discount the future payments at the EAR. Since YTM < EAR, discounting at the YTM and thus selling the bond will result in a higher present value.

Since the second project has a profitability index of 2.2, the present value of the future cash flows is ($1000)(2.2) = $2,200. Thus, $NPV_2 = 2200 - 1000 = 1200$. Since $NPV_2 > NPV_1$, the firm should choose the second project.

3.

a. The statement is false. Here's a counter-example: Let the tax rate be 35%. Then, the net income will decrease by $1,000,000 - 1,000,000 \cdot 0.35 = 1,000,000 - 350,000 = 650,000$. $350,000 is the tax savings because of the increase in the interest paid.

b. The relevant equation is:

   Loan amount - closing costs - fees = Present value of the monthly payments at APR .

   If closing costs increase, then the left-hand-side will decrease which means the monthly payments should be discounted at a higher rate. Thus, APR should be higher. The statement is true.

c. The higher the coupon rate, the lower the interest rate risk. Since duration is a direct measure of interest rate risk, it should be lower. The statement is thus false.

d. The statement is false. A counter example is (-1000, 1000) which has a payback period of 1 year, but for all positive interest rates the NPV is less than zero.

e. This statement is false. A counter example is given below:

   (-1, 2) has an IRR of 100%, whereas (-1000000, 1999000) has an IRR of 99.9%. Clearly, for reasonable discount rates the second project has a higher NPV (for instance for r very close to 0, first one has an NPV of about $1 whereas the second one has an NPV of about $999,000).