Midterm Fall 2008 Solutions

1. a. We first find the quarterly rate. Since the interest is compounded monthly, the effective monthly rate is $12\%/12 = 1\%$. Thus, the effective quarterly rate is $(1+0.01)^3 - 1 = 0.0303$. EAR $= 1.01^{12} - 1 = 12.68\%$

The present value of the benefits for one employee at her retirement day is:

$$PV_i = 10,000 + \frac{10,000 \cdot 1.02}{0.0303 - 0.02}\left(1 - \left(\frac{1.02}{1.0303}\right)^{79}\right) = 552,514.02 \tag{0.1}$$

Now, we have an annuity with 10 payments of \$552,514.02 (one for each employee at the corresponding retirement day) with the first payment on Jan 1 2015. The PV of this annuity on Jan 1 2009 is:

$$PV = \frac{552,514.02}{0.1268}\left(1 - \frac{1}{1.1268^{10}}\right)\frac{1}{1.1268^5} = 1,671,444.39. \tag{0.2}$$

b. We need to calculate the present value of the future payments to find the price of the bond. We use the discount rates implied by the zero-coupon bonds:

$$925.93 = \frac{1000}{1+r_1} \tag{0.3}$$

$$889.99 = \frac{1000}{(1+r_2)^2} \tag{0.4}$$

$$925.93 = \frac{1000}{(1+r_3)^3} \tag{0.5}$$

$$\tag{0.6}$$

Price of each bond is equal to:

$$Price = \frac{80}{1+r_1} + \frac{80}{(1+r_2)^2} + \frac{1080}{(1+r_3)^3} \tag{0.7}$$

$$= \frac{80}{1000/925.93} + \frac{80}{1000/889.99} + \frac{1080}{1000/751.31} = 956.69 \tag{0.8}$$

Thus, she will receive 275 x 956.69 = 263,089.31. The PV of the benefits is $552,514.02/(1.1268^6) = 269,900.94$. So, she should reject the bonds.

2. *Initial layout* $= \$1,500,000 + \$400,000 = \$1,900,000.$
Annual depreciation for years 1-6 $= \$1,500,000/6 = \$250,000.$
Annual depreciation for years 7-12 $= 0.$
OCF for years 1-6 $= (100,000 - (-250,000) - 250,000)(1 - 0.40) + 250,000 = 310,000.$
OCF for years 7-12 $= (100,000 - (-250,000) - 0)(1 - 0.40) + 0 = 210,000.$
Terminal non-oper. CF $= 500,000 + 400,000 - 0.40(500,000 - 0) = 700,000.$

$$NPV = -1,900,000 + \frac{445,000}{0.12}\left(1 - \frac{1}{1.12^6}\right) + \frac{210,000}{0.12}\left(1 - \frac{1}{1.12^6}\right)\frac{1}{1.12^6} + \frac{700,000}{1.12^{12}} = -8,368.12$$

3.

a. False. Duration is a direct measure of interest rate risk. Since both bonds have the same duration, they have the same interest rate risk.

b. True. Let the discounted payback period be t years. This means the present value of the cash flows up to and including year t is equal to the initial cost. However, we have that the sum of positive cash flows is greater than or equal to the present value of the same cash flows because of discounting. Thus, the sum of the cash flows up to and including year t is greater than or equal to the initial cost which means the payback period is less than t years. (I will also accept the following answer although it is not technically correct: False because the cash flow might not have a discounted payback period/payback period).

c. False. Loan amount $= \dfrac{\text{Montly payment}}{rate/12}\left(1 - \dfrac{1}{(1+rate/12)^T}\right)$. It is given that the loan amounts and the rates are the same. Thus, monthly payments must be the same using this equation.

d. False. The current stock value reflects the risk, timing and magnitude of all future cash flows, both short-term and long-term.

e. True. Since the cash flow is conventional and the IRR is greater than the required rate of return, NPV must be positive. This implies that the present value of future cash flows during the lifetime of the project is greater than the initial cost. But this means project pays back (in the sense of discounted payback) at some point before the end of the project.