E-120 Principles of Engineering Economics
Fall 06 Midterm I Solutions

1. Let's find the present value of each cash flow on Jan 01 2007 (any other date is also fine since it will not change your final decision).

Payment 1:

The relevant cash flow is,

$$\left(150000,\ 150000(1.02),\ 150000(1.02)^2, \ldots\ldots, 150000(1.02)^{18}\right)$$

where each period corresponds to 6 months. Since YTM is compounded semiannually, the effective rate for period is 4%.

The corresponding present value is:

$$PV = \sum_{i=1}^{19} \frac{150000(1.02)^{i-1}}{1.04^{i-1}} = 150000\frac{1.04}{1.02}\sum_{i=1}^{19}\frac{1}{(1.04/1.02)^i}$$

$$= 150000\frac{1.04}{1.02}\frac{1-\left(\frac{1.04}{1.02}\right)^{-19}}{\frac{1.04}{1.02}-1} = \mathbf{2406577.787}$$

Payment 2:

The relevant cash flow is,

$$\left(200000,\ 500000,\ 200000,\ 500000, 200000, 500000,\ 200000,\ 500000,\ 200000,\ 500000\right)$$

where each period corresponds to 1 year. The EAR is $1.04^2 - 1 = 8.16\%$ and the effective 2-year rate is $1.04^4 - 1 = 16.99\%$. So the present value becomes:

$$PV = 200000 + 200000\frac{1-1.1699^{-4}}{0.1699} + \left(500000\frac{1-1.1699^{-5}}{0.1699}\right)(1.0816) = \mathbf{2479568.98}$$

.

Here, the payments of \$200000 might be considered as an annuity due where 1 period corresponds to one year.

So, we choose payment plan 2.

2.

(a) We need to find the present value of the cash flows on Jan 01 2006:

$$PV = \sum_{i=1}^{15} \frac{40}{(1.04)^{i+5}} + \frac{225}{(1.04)^6} + \frac{1000}{(1.04)^{20}} = \frac{1}{1.04^5} \sum_{i=1}^{15} \frac{40}{(1.04)^i} + 634.21$$

$$= \frac{1}{1.04^5} 40 \frac{1-1.04^{-15}}{0.04} + 634.21 = \mathbf{999.747875} \cong \mathbf{1000}$$

(b) $F = 999.75(1.04)^{20} = \mathbf{2190.57}$

(If you assume YTM is compounded annually, then
$F = 999.75(1.08)^{10} = \mathbf{2158.38}$).

3.

(a) TRUE $f_{0,2} = s_2$ by definition. We also have:

$$(1+f_{0,2})^2(1+f_{2,3}) = (1+s_3)^3 \Rightarrow (1+s_2)^2(1+f_{2,3}) = (1+s_3)^3 \text{ since } f_{0,2} = s_2$$

$$\Rightarrow \frac{(1+s_3)^3}{(1+s_2)^2} = (1+f_{2,3}) \Rightarrow \frac{(1+s_3)^3}{(1+s_2)^3} = \frac{(1+f_{2,3})}{(1+s_2)} < 1 \text{ since } f_{0,2} = s_2 > f_{2,3} \Rightarrow s_3 < s_{2.}$$

(b) TRUE Present value of the payments at the quote rate is equal to the loan amount L. However, the present value of the payments at the APR is lower than the loan amount since the closing costs and the points are positive. Therefore, since the payments are the same no matter which rate you are using, APR must be less than the quote rate (the higher the rate, the lower the present value).

(c) FALSE Future value of the first cash flow is equal to the present value of it compounded by the appropriate number of periods using the interest rate *r*.

However, the second cash flow also has the same PV as the first one, and moreover the discount rate used and the number of periods are also the same. Thus, they will always have the same future values at some time *t*.

(One might argue that it's not mentioned how the interest rate is compounded and that if the compounding is different, then the claim might be true. If you did so, you should have got full credit).

(d) FALSE The maturity of the coupon-bond might be chosen to be sufficiently high to make its duration larger than the duration of the zero coupon bond.

(e) FALSE If the first dividend of the company B is high enough its price might be higher than the price of the company B.